Υπερβολικές Συναρτήσεις

1. Ορισμοί

Υπερβολικό Ημίτονο: $\sinh x = \frac{1}{2} \cdot (e^x - e^{-x})$

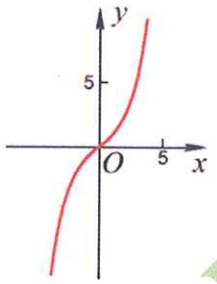
Υπερβολικό Συνημίτονο: $\cosh x = \frac{1}{2} \cdot (e^x + e^{-x})$

Υπερβολική Εφαπτομένη: $\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

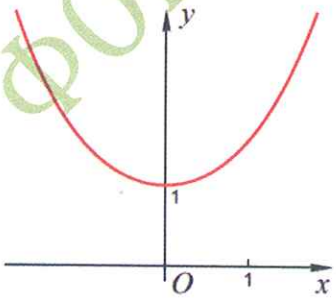
Υπερβολική Συνεφαπτομένη: $\coth x = \frac{\cosh x}{\sinh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$.

Η Υπερβολική Τέμνουσα ($\operatorname{sech} x = 1/\cosh x$) και Υπερβολική Συντέμνουσα ($\operatorname{csch} x = 1/\sinh x$) δε χρησιμοποιούνται συχνά.

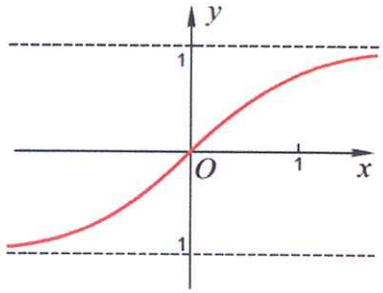
2. Γραφικές Παραστάσεις



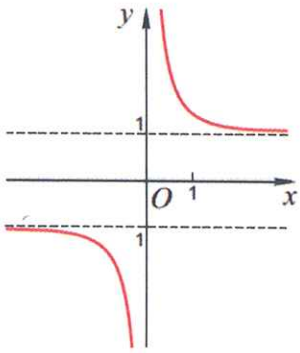
$y = \sinh x$



$y = \cosh x$



$y = \tanh x$



$y = \coth x$

3. Ταυτότητες

$\cosh^2 x - \sinh^2 x = 1$ $\sinh x + \cosh x = \dfrac{1}{\cosh x - \sinh x}$ $(\sinh x + \cosh x)^n = \sinh(n \cdot x) + \cosh(n \cdot x)$
$\sinh(-x) = -\sinh x$ $\cosh(-x) = \cosh x$ $\tanh(-x) = -\tanh x$ $\coth(-x) = -\coth x$
$\sinh(x - y) = \sinh x \cdot \cosh y - \cosh x \cdot \sinh y$ $\cosh(x - y) = \cosh x \cdot \cosh y - \sinh x \cdot \sinh y$ $\tanh(x - y) = \dfrac{\tanh x - \tanh y}{1 - \tanh x \cdot \tanh y}$ $\coth(x - y) = \dfrac{\coth x \cdot \coth y - 1}{\coth x - \coth y}$

$\sinh(x+y) = \sinh x \cdot \cosh y + \cosh x \cdot \sinh y$ $\cosh(x+y) = \cosh x \cdot \cosh y + \sinh x \cdot \sinh y$ $\tanh(x+y) = \dfrac{\tanh x + \tanh y}{1 + \tanh x \cdot \tanh y}$ $\coth(x+y) = \dfrac{\coth x \cdot \coth y + 1}{\coth x + \coth y}$
$\sinh x - \sinh y = 2 \cdot \cosh\left(\dfrac{x+y}{2}\right) \cdot \sinh\left(\dfrac{x-y}{2}\right)$ $\cosh x - \cosh y = 2 \cdot \sinh\left(\dfrac{x+y}{2}\right) \cdot \sinh\left(\dfrac{x-y}{2}\right)$ $\tanh x - \tanh y = \dfrac{\sinh(x-y)}{\cosh x \cdot \cosh y}$
$\sinh x + \sinh y = 2 \cdot \sinh\left(\dfrac{x+y}{2}\right) \cdot \cosh\left(\dfrac{x-y}{2}\right)$ $\cosh x + \cosh y = 2 \cdot \cosh\left(\dfrac{x+y}{2}\right) \cdot \cosh\left(\dfrac{x-y}{2}\right)$ $\tanh x + \tanh y = \dfrac{\sinh(x+y)}{\cosh x \cdot \cosh y}$
$\sinh x \cdot \sinh y = \dfrac{1}{2} \cdot [\cosh(x-y) - \cosh(x+y)]$ $\cosh x \cdot \cosh y = \dfrac{1}{2} \cdot [\cosh(x-y) + \cosh(x+y)]$ $\sinh x \cdot \cosh y = \dfrac{1}{2} \cdot [\sinh(x-y) + \sinh(x+y)]$
$\sinh^2 x - \sinh^2 y = \sinh(x+y) \cdot \sinh(x-y)$ $\sinh^2 x + \cosh^2 y = \cosh(x+y) \cdot \cosh(x-y)$
$\sinh(2 \cdot x) = 2 \cdot \sinh x \cdot \cosh x$ $\cosh(2 \cdot x) = \cosh^2 x + \sinh^2 x = 2 \cdot \cosh^2 x - 1 = 1 - 2 \cdot \sinh^2 x$ $\tanh(2 \cdot x) = \dfrac{2 \cdot \tanh x}{1 + \tanh^2 x}$ $\coth(2 \cdot x) = \dfrac{\coth^2 x + 1}{2 \cdot \coth x}$
$\tanh x = \dfrac{\cosh(2 \cdot x) - 1}{\sinh(2 \cdot x)} = \dfrac{\sinh(2 \cdot x)}{\cosh(2 \cdot x) + 1}$
$\sinh(3 \cdot x) = 3 \cdot \sinh x + 4 \cdot \sinh^3 x$ $\cosh(3 \cdot x) = -3 \cdot \cosh x + 4 \cdot \cosh^3 x$ $\tanh(3 \cdot x) = \dfrac{3 \cdot \tanh x + \tanh^3 x}{1 + 3 \cdot \tanh^2 x}$ $\coth(3 \cdot x) = \dfrac{3 \cdot \coth x + \coth^3 x}{1 + 3 \cdot \coth^2 x}$

$\sinh(4\cdot x) = 4\cdot\sinh x\cdot\cosh x + 8\cdot\sinh^3 x\cdot\cosh x$ $\cosh(4\cdot x) = -8\cdot\cosh^2 x + 8\cdot\cosh^4 x + 1$ $\tanh(4\cdot x) = \dfrac{4\cdot\tanh x + 4\cdot\tanh^3 x}{1 + 6\cdot\tan^2 x + \tan^4 x}$
$\sinh^2 x = \dfrac{-1 + \cosh(2\cdot x)}{2}$ $\cosh^2 x = \dfrac{1 + \cosh(2\cdot x)}{2}$
$\sinh^3 x = \dfrac{-3\cdot\sinh x + \sinh(3\cdot x)}{4}$ $\cosh^3 x = \dfrac{3\cdot\cosh x + \cosh(3\cdot x)}{4}$
$\sinh^4 x = \dfrac{3}{8} - \dfrac{1}{2}\cdot\cosh(2\cdot x) + \dfrac{1}{8}\cdot\cosh(4\cdot x)$ $\cos^4 x = \dfrac{3}{8} + \dfrac{1}{2}\cdot\cosh(2\cdot x) + \dfrac{1}{8}\cdot\cosh(4\cdot x)$

4. Σχέσεις με Τριγωνομετρικές Συναρτήσεις

$\sin(i\cdot x) = i\cdot\sinh x$ $\cos(i\cdot x) = \cosh x$ $\tan(i\cdot x) = i\cdot\tanh x$ $\cot(i\cdot x) = -i\cdot\coth x$
$\sinh(i\cdot x) = i\cdot\sin x$ $\cosh(i\cdot x) = \cos x$ $\tanh(i\cdot x) = i\cdot\tan x$ $\coth(i\cdot x) = -i\cdot\cot x$
$\sinh(x - i\cdot y) = \sinh x\cdot\cos y - i\cdot\cosh x\cdot\sin y$ $\cosh(x - i\cdot y) = \cosh x\cdot\cos y - \sinh x\cdot\sin y$ $\tanh(x - i\cdot y) = \dfrac{\sinh(2\cdot x) - i\cdot\sin(2\cdot y)}{\cosh(2\cdot x) + \cos(2\cdot y)}$ $\coth(x - i\cdot y) = \dfrac{\sinh(2\cdot x) + i\cdot\sin(2\cdot y)}{\cosh(2\cdot x) - \cos(2\cdot y)}$
$\sinh(x + i\cdot y) = \sinh x\cdot\cos y + i\cdot\cosh x\cdot\sin y$ $\cosh(x + i\cdot y) = \cosh x\cdot\cos y + \sinh x\cdot\sin y$ $\tanh(x + i\cdot y) = \dfrac{\sinh(2\cdot x) + i\cdot\sin(2\cdot y)}{\cosh(2\cdot x) + \cos(2\cdot y)}$ $\coth(x - i\cdot y) = \dfrac{\sinh(2\cdot x) - i\cdot\sin(2\cdot y)}{\cosh(2\cdot x) - \cos(2\cdot y)}$